

Fig. 11
Fig. 11 shows a sketch of the curve with equation $y=x-\frac{4}{x^{2}}$.
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ and show that $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=-\frac{24}{x^{4}}$.
(ii) Hence find the coordinates of the stationary point on the curve. Verify that the stationary point is a maximum.
(iii) Find the equation of the normal to the curve when $x=-1$. Give your answer in the form $a x+b y+c=0$.

2 (i) Use calculus to find, correct to 1 decimal place, the coordinates of the turning points of the curve $y=x^{3}-5 x$. [You need not determine the nature of the turning points.]
(ii) Find the coordinates of the points where the curve $y=x^{3}-5 x$ meets the axes and sketch the curve. [4]
(iii) Find the equation of the tangent to the curve $y=x^{3}-5 x$ at the point $(1,-4)$. Show that, where this tangent meets the curve again, the $x$-coordinate satisfies the equation

$$
x^{3}-3 x+2=0
$$

Hence find the $x$-coordinate of the point where this tangent meets the curve again.

3 A cubic curve has equation $y=x^{3}-3 x^{2}+1$.
(i) Use calculus to find the coordinates of the turning points on this curve. Determine the nature of these turning points.
(ii) Show that the tangent to the curve at the point where $x=-1$ has gradient 9 .

Find the coordinates of the other point, P , on the curve at which the tangent has gradient 9 and find the equation of the normal to the curve at P .

Show that the area of the triangle bounded by the normal at P and the $x$ - and $y$-axes is 8 square units.

Fig. 10 shows a sketch of the graph of $y=7 x-x^{2}-6$.


Fig. 10
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ and hence find the equation of the tangent to the curve at the point on the curve where $x=2$.

Show that this tangent crosses the $x$-axis where $x=\frac{2}{3}$.
(ii) Show that the curve crosses the $x$-axis where $x=1$ and find the $x$-coordinate of the other point of intersection of the curve with the $x$-axis.
(iii) Find $\int_{1}^{2}\left(7 x-x^{2}-6\right) \mathrm{d} x$.

Hence find the area of the region bounded by the curve, the tangent and the $x$-axis, shown shaded on Fig. 10.


Fig. 11
The equation of the curve shown in Fig. 11 is $y=x^{3}-6 x+2$.
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(ii) Find, in exact form, the range of values of $x$ for which $x^{3}-6 x+2$ is a decreasing function.
(iii) Find the equation of the tangent to the curve at the point $(-1,7)$.

Find also the coordinates of the point where this tangent crosses the curve again.

